CORRELATION FUNCTIONS OF INTERNAL STRESSES OF DISLOCATION LOOPS

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The internal stresses in face-centered cubic monocrystals associated with prismatic dislocation loops are calculated, based on nonsimultaneous equations. Assuming that the Burgers vector of each loop has equiprobable orientations coordinated with the system of sliding planes and that the loops themselves are distributed randomly in space, the characteristics of a random component of the internal field of stresses are calculated. The spectral density of the energy and the binary correlation function of the tensors of the internal stresses are found. The tensor and coordinate relationships of the correlation functions are analyzed.

The internal stresses considerably influence the physical and mechanical properties of solids. Various defects in the structure can be the cause of their occurrence. However, dislocations are the biggest contributing factor to internal stresses.

Dislocation stresses are determined by the dislocation density and the structure of the dislocation networks. A large number of experimental works [1] are devoted to investigation of these. However, success has been achieved in carrying out theoretical calculation in a limited number of cases especially in conformity with the parallel arrangement of the lines of dislocation. Hence such integral characteristics as the dispersion and energy of the internal stresses [2–4] were calculated. Although this approach gives certain information about internal stresses, it has the disadvantage that it does not enable their three-dimensional distribution to be evaluated.

On the other hand, the elastic field of dislocation clusters has been investigated in detail for a number of regular dislocation structures: boundaries of blocks, polygonal walls, and clusters of rectilinear dislocations [5]. An obvious drawback of the model approach for integral description of the field of internal stresses will be, on the one hand, the great variety of dislocation structures and, on the other hand, the irregularity of the distribution of the dislocations.

Beginning with this, it is of interest to calculate the internal stresses associated with dislocations, based on the theory of random functions. It is convenient to adopt binary correlation functions of internal stresses as the integral characteristic of the elastic field of the system of dislocations; from these, such parameters as the dispersion and the internal energy can be obtained. These functions also describe the three-dimensional decrease of the correlation connections of the random field of internal stresses.

1. In the continuum theory of dislocations the sources of internal stresses are characterized by the nonsimultaneity tensor η_{mn}, and the internal stresses σ_{kl} are determined from the solution of the following system of equations:

$$\mathrm{Rot}_{nj}^{mi} s_{ijkl}\sigma_{kl} = \eta_{mn}, \quad \nabla_l \sigma_{kl} = 0, \quad \mathrm{Rot}_{nj}^{mi} \equiv \varepsilon_{ipm}\varepsilon_{jqn}\nabla_p\nabla_q \tag{1.1}$$

with corresponding boundary conditions [6].

Here s_{ijkl} is the tensor of elastic ductilities, ε_{ipm} is the unit antisymmetrical tensor, ∇_i indicates differentiation with respect to the corresponding coordinate, and summation is assumed according to the repeating indices.

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The distribution of dislocations in a real crystal can be characterized by the tensor of the dislocation density α_{ik} or by the tensor of the density of the dislocation moments μ_{ik} (dislocation polarization), which is associated with the tensor of nonsimultaneity of the relationships

$$\eta_{ij} = \nabla_k \varepsilon_{kl(i} \alpha_{j)l}, \qquad \alpha_{jl} = \varepsilon_{jmn} \nabla_m \mu_{nl} \tag{1.2}$$

Here symmetrization is carried out according to the indices included in the parentheses.

It follows from the homogeneity of the tensor of ductilities that the regular $\langle \sigma_{kl} \rangle$ and random σ'_{kl} components of the field of stresses are determined by the equations

$$\mathrm{Rot}^{mi}_{nj} s_{ijkl} \langle \sigma_{kl} \rangle = \langle \eta_{mn} \rangle, \qquad \mathrm{Rot}^{mi}_{nj} s_{ijkl} \sigma'_{kl} = \eta'_{mn} \tag{1.3}$$

Here and further on, the angular brackets are used to indicate statistical averaging, and the random components of the corresponding magnitudes are designated by primes.

We will consider that the random fields of the dislocation density and those of the dislocation polarization are statistically homogeneous. Their mean values will then be constant over the crystal, and the mean values of the tensors of nonsimultaneity and of the internal stresses will be equal to zero in accordance with Eqs. (1.1) and (1.2). As a result of statistical homogeneity the binary correlation functions will only depend on the radius vector $\mathbf{r}$ which connects the two examined points of the crystal:

$$M^{kl}_{ij}(\mathbf{r}) = \langle \mu'_{ij}(\mathbf{r}_1) \mu'_{kl}(\mathbf{r}_1 + \mathbf{r}) \rangle, \qquad A^{kl}_{ij}(\mathbf{r}) = \langle \alpha'_{ij}(\mathbf{r}_1) \alpha'_{kl}(\mathbf{r}_1 + \mathbf{r}) \rangle \tag{1.4}$$

The correlation function $H^{kl}_{ij}(\mathbf{r})$ of the tensor of nonsimultaneity can be expressed through the functions A^{kl}_{ij} and M^{kl}_{ij}:

$$H^{kl}_{ij} = -\mathrm{Rot}^{p(i}_{q(k} A^{j)p}_{l)q} = \mathrm{Rot}^{p(i}_{q(k} \mathrm{Rot}^{j)n}_{l)m} M^{np}_{mq} \tag{1.5}$$

The solution of system (1.1) for an unrestricted crystal is presented in the form of a convolution integral with the Green tensor G^{pq}_{kl} of the internal stresses:

$$\sigma_{kl}(\mathbf{r}) = \int G^{pq}_{kl}(\mathbf{r} - \boldsymbol{\rho}) \eta_{pq}(\boldsymbol{\rho}) \, d\boldsymbol{\rho}, \qquad d\boldsymbol{\rho} \equiv d\rho_x d\rho_y d\rho_z \tag{1.6}$$

in which the Green tensor is determined by the equations [6]

$$\mathrm{Rot}^{mi}_{nj} s_{ijkl} G^{pq}_{kl} = -\frac{1}{8\pi} \mathrm{Rot}^{mr}_{ns} \mathrm{Rot}^{rp}_{sq} r, \qquad \nabla_q G^{pq}_{kl} = 0 \tag{1.7}$$

The second equation of (1.7) is a condition of bivortical nature of the elastic fields of the internal stresses.

The expression (1.6) enables the following integral representation of the binary correlation function of the tensors of the internal stresses Σ^{kl}_{ij} to be obtained:

$$\Sigma^{kl}_{ij}(\mathbf{r}) = \iint G^{pq}_{ij}(\mathbf{r}_1 - \boldsymbol{\rho}_1) H^{pq}_{mn}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) G^{kl}_{mn}(\mathbf{r}_2 - \boldsymbol{\rho}_2) \, d\boldsymbol{\rho}_1 d\boldsymbol{\rho}_2, \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2 \tag{1.8}$$

Changing over in expression (1.8) to the Fourier transforms, we find

$$\Sigma^{kl}_{ij}(\mathbf{r}) = \frac{1}{8\pi^3} \int G^{pq}_{ij}(\mathbf{k}) H^{pq}_{mn}(\mathbf{k}) G^{kl}_{mn}(-\mathbf{k}) e^{i\mathbf{kr}} \, d\mathbf{k} \tag{1.9}$$

$$\Sigma^{kl}_{ij}(\mathbf{k}) = G^{pq}_{ij}(\mathbf{k}) H^{pq}_{mn}(\mathbf{k}) G^{kl}_{mn}(-\mathbf{k}) \tag{1.10}$$

Here the integral Fourier transform used and the integral representation of the δ-function are

$$G(\mathbf{r}) = \frac{1}{8\pi^3} \int G(\mathbf{k}) e^{i\mathbf{kr}} \, d\mathbf{k}, \quad \delta(\mathbf{r}) = \frac{1}{8\pi^3} \int e^{i\mathbf{kr}} \, d\mathbf{k} \tag{1.11}$$

The expressions (1.9) and (1.10) together with the relationships (1.5) give a general solution of the set problem concerning the connection between the correlation function of the tensor of the internal stresses and the correlation functions of the tensor of the density of dislocations and the dislocation polarization.

TABLE 1

	G, 10^{12} dyn/cm^2	m	p_{11}, 10^{12} dyn/cm^2	p_{12}, 10^{12} dyn/cm^2	p_{44}, 10^{12} dyn/cm^2	u, 10^{12} dyn/cm^2
Al	0.26	1.54	0.46	0.45	0.10	0.23
Cu	0.55	1.48	0.94	0.80	0.22	0.56
Ge	0.56	1.24	0.79	0.74	0.22	0.59
Si	0.68	1.28	0.97	0.92	0.26	0.65

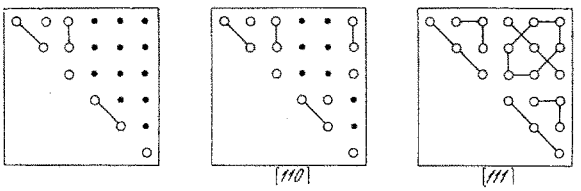

Fig. 1

We will subsequently adopt an approximation of the isotropy of the elastic moduli [5]; however, we will consider that the distribution of dislocations in the crystal are in agreement with the crystallographic axes. Then from Eqs. (1.7) we obtain an explicit form of the Fourier transform of the Green tensor [7]:

$$G_{ij}^{pq}(\mathbf{k}) = \frac{2G}{k^2}[m\xi_{ij}\xi_{pq} - \xi_{p(i}\xi_{j)q}], \qquad m \equiv \frac{2(3K+G)}{3K+4G} \tag{1.12}$$
$$\xi_{ij} \equiv \frac{k_i k_j}{k^2} - \delta_{ij}$$

which enables the function Σ_{ij}^{kl} to be represented in the following form:

$$\Sigma_{ij}^{kl}(\mathbf{k}) = \frac{4G^2}{k^4}[\xi_{p(i}\xi_{j)q} - m\xi_{ij}\xi_{pq}][\xi_{k(m}\xi_{n)l} - m\xi_{mn}\xi_{kl}]\, H_{mn}^{pq}(\mathbf{k}) \tag{1.13}$$

Here K and G are the mean moduli of compression on all sides and shear.

2. We will adapt the results obtained to monocrystals containing nonoverlapping dislocation loops. For definiteness we will examine the whole of the prismatic dislocations in face-centered cubic crystals. Dislocations of this kind are plates of vacancies or of introduced atoms [8] whose thickness corresponds in order of magnitude to a constant lattice. Electron-microscopic investigation of a number of face-centered cubic crystals [8-12] shows that the prismatic dislocations lie in the $\{110\}$ planes with the Burgers vectors $b \approx 1/2\ \langle 110\rangle$, which are oriented normal to the plane of the loop.

The distribution of the dislocation loops will be characterized by the tensor of dislocation polarization. For the dislocation discs examined, this tensor can be represented as follows:

$$\mu_{ik} = b_i n_k h^{-1}\delta(V) \tag{2.1}$$

where $\delta(V)$ is the δ-function of the region V which is occupied by the dislocation disc [13] and $\mathbf{n}$ is the unit vector of the normal to its plane.

We will consider that the loops are scattered in a chaotic manner over the volume of the crystal and their Burgers vectors are equally likely to have any $\langle 110\rangle$ direction. Then $\langle \mu_{ik}\rangle = 0$.

In order to determine the dispersion $\Delta_{ij}^{kl} \equiv M_{ij}^{kl}(0)$ of the random field of the tensor of the dislocation polarization, we will carry out averaging in two stages. In the first place we will find the arithmetical mean $\langle \mu_{ij}(\mathbf{r})\mu_{kl}(\mathbf{r})\rangle_\omega$. The index ω represents averaging over all 12 orientations of the Burgers vector. In the crystallographic system of coordinates we have

$$\langle \mu_{ij}(\mathbf{r})\,\mu_{kl}(\mathbf{r})\rangle_\omega = \frac{b^2}{12h^2}\Big[\delta_{ij}\delta_{kl} + 2\delta_{i(k}\delta_{l)j} - \sum_{n=1}^{3}\delta_{in}\delta_{jn}\delta_{kn}\delta_{ln}\Big]\delta(V) \tag{2.2}$$

We will now average (2.2) with respect to the crystal volume; this gives

$$\Delta_{ij}^{kl} = \frac{\gamma b^2}{12h^2}\Big[\delta_{ij}\delta_{kl} + 2\delta_{i(k}\delta_{l)j} - \sum_{n=1}^{3}\delta_{in}\delta_{jn}\delta_{kn}\delta_{ln}\Big] \tag{2.3}$$

where γ is the relative volume occupied by the dislocation plates.

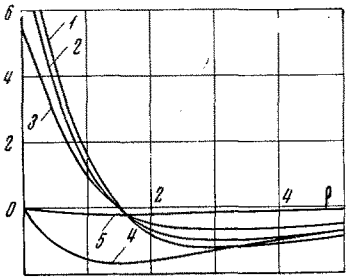

Fig. 2

We will present the correlation function M_{ij}^{kl} as a product of the tensor Δ_{ij}^{kl} and coordinate φ (r) components:

$$M_{ij}^{kl}(\mathbf{r}) = \Delta_{ij}^{kl}\,\varphi(\mathbf{r}) \tag{2.4}$$

The function φ (r) is determined by the law of distribution of dislocation loops over the volume of the crystal. The expression (2.4) together with (1.5) and (1.13) enables the correlation tensor of internal stresses to be represented in the form

$$\Sigma_{ij}^{kl}(\mathbf{r}) = 4G^2\psi_{ij}^{pq}\psi_{mn}^{kl}e_{vam}e_{sbp}e_{nct}e_{qdr}\Delta_{rs}^{tv}I_{ab}^{cd}(\mathbf{r}),\quad \psi_{mn}^{kl} \equiv m\delta_{mn}\delta_{kl} - \delta_{k(m}\delta_{n)l} \tag{2.5}$$

$$I_{ij}^{kl} \equiv \nabla_i\nabla_j\nabla_k\nabla_l I(\mathbf{r}),\quad I_i(\mathbf{r}) = \frac{1}{8\pi^3}\int\frac{\varphi(\mathbf{k})}{k^4}e^{i\mathbf{kr}}\,d\mathbf{k} \tag{2.6}$$

First of all we will calculate the dispersion of the internal stresses. For this purpose we bear in mind that $\varphi(0) \equiv 1$ and also the relationships

$$\int_0^\infty \varphi(k)\,k^2dk = 2\pi^2\varphi(0),\qquad \int n_in_jn_kn_l\,d\Omega = \frac{4\pi}{15}\delta_{ijkl}$$
$$\delta_{ijkl} \equiv \delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk},\qquad n_i \equiv k_i/k \tag{2.7}$$

Then, by changing over to spherical coordinates in (2.6) we find

$$I_{ij}^{kl}(0) = {}^1/_{15}\,\delta_{ijkl} \tag{2.8}$$

Hence

$$\Sigma_{ij}^{kl}(0) = \frac{g^2}{45}\left[\nu\delta_{ij}\delta_{kl} + 14\,\delta_{k(i}\delta_{j)l} - 3\sum_{n=1}^{3}\delta_{in}\delta_{jn}\delta_{kn}\delta_{ln}\right]$$
$$g^2 \equiv \frac{\gamma b^2}{h^2}G^2,\qquad \nu \equiv 96\,m^2 - 64\,m + 7 \tag{2.9}$$

It follows from (2.9) that the autocorrelation tensor of the internal stresses has cubic symmetry. By changing over from tensor designations to matrix designations, we will quote three independent components of this magnitude:

$$\Sigma_{11} = \frac{2g^2}{45}(48\,m^2 - 32\,m + 9),\qquad \Sigma_{12} = \frac{\nu g^2}{45},\quad \Sigma_{44} = \frac{7g^2}{45} \tag{2.10}$$

The density of the internal energy associated with dislocations can be found according to the known dispersion of internal stresses:

$$U = {}^1/_2\,s_{ijkl}\,\Sigma_{ij}^{kl}(0) \tag{2.11}$$

Substituting here the value $\Sigma_{ij}^{kl}(0)$ in accordance with (2.9), we obtain

$$U = {}^1/_{30}g^2\left[\nu\,(s_{11} + 2s_{12}) + 11s_{11} + 7s_{44}\right] \tag{2.12}$$

The energy characteristic of nonuniformity of distribution of internal stresses can be the spectral density of energy E (k). The latter is determined by the relationship [14]

$$U = \int_0^\infty E(k)\,dk \tag{2.13}$$

The magnitude E(k) is connected in the following way with the Fourier transform of the correlation tensor of internal stresses:

$$E(k) = \frac{k^2}{16\pi^3}s_{ijkl}\int\Sigma_{ijkl}(\mathbf{k})\,d\Omega \tag{2.14}$$

Hence, by using Eqs. (2.5) and (2.6) we obtain

$$E(k) = \frac{1}{2\pi^2}Uk^2\varphi(k) = \frac{4}{\pi}a^3Uk^2\,(1+a^2k^2)^{-2} \tag{2.15}$$

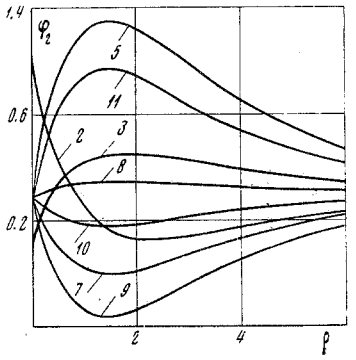

Fig. 3

where the coordinate relationship of the correlation tensor is taken as exponential with the correlation scale a

$$\varphi(r) = \exp(-r/a), \qquad \varphi(k) = 8\pi a^3 (1 + a^2k^2)^{-2} \tag{2.16}$$

It follows from Eq. (2.15) that for long waves $E(k) \sim k^2$, while for short waves $E(k) \sim k^{-2}$. The maximum $E(k)$ matches the value $k = a^{-1}$.

In order to evaluate the influence of the material on Σ and U we will introduce new parameters with the equations

$$u = U/\gamma, \qquad p_{mn} = (\gamma^{-1}\Sigma_{mn})^{1/2}$$

The values of these parameters for four materials are given in Table 1. In order to calculate u and p_{mn} it was assumed that the thickness of the dislocation plates was equal to the Burgers vector. It is seen from Table 1 that at the same density of the dislocation loops, silicon has the highest energy, and aluminum the lowest.

In order to evaluate the parameter γ, we will take into account the fact that in aluminum and copper the diameter of the dislocation loops $\sim 10^2$ Å and their density is $\sim 10^{14}$ cm^{-3} [5]. Hence $\gamma \sim 10^{-6}$, which corresponds with the concentration of quenching vacancies. Having this in view we obtain

$$\Sigma_{11}^{1/2} = 5 \text{ kgf/mm}^2, \qquad \Sigma_{44}^{1/2} = 1 \text{ kgf/mm}^2, \qquad U = 2\cdot 10^5 \text{ ergs/cm}^3,$$

for aluminum.

3. The dispersion and internal energy of the dislocation stresses were calculated above. We will now turn to the finding of a binary correlation function of dislocation stresses.

Calculation of the integrals introduced by Eqs. (2.6) in the case of conditions (2.16) gives

$$I(\mathbf{r}) = a^4 [(1 + 4\rho^{-1}) e^{-\rho} - 4\rho^{-1}(1 + {}^1/_4\rho^2)], \quad \rho \equiv r/a \tag{3.1}$$

$$I_{ij}^{kl}(\mathbf{r}) = f_1(\rho)\,\delta_{ijkl} + f_2(\rho)\,n_in_jn_kn_l + f_3(\rho)\,P_{ijkl}$$

$$P_{ijkl} \equiv 2(\delta_{[ij}n_kn_{l]} + \delta_{[ik}n_jn_{l]} + \delta_{[il}n_jn_{k]}) \tag{3.2}$$

$$\rho^5 f_1 \equiv -12 + \rho^2 + (12 + 12\rho + 5\rho^2 + \rho^3) e^{-\rho}$$

$$\rho^5 f_2 \equiv 3(100 - 7\rho^2) - (300 + 300\rho + 129\rho^2 + 29\rho^3 + 2\rho^4 - \rho^5) e^{-\rho}$$

$$\rho^5 f_3 \equiv 3(20 - \rho^2) - (60 + 60\rho + 27\rho^2 + 7\rho^3 + \rho^4) e^{-\rho} \tag{3.3}$$

The square brackets used in Eq. (3.2) indicate symmetrization according to pairs of indices for example:

$$\delta_{[ij}n_kn_{l]} \equiv {}^1/_2(\delta_{ij}n_kn_l + \delta_{kl}n_in_j) \tag{3.4}$$

Substituting the expressions (2.3), (3.2), and (3.3) into (2.5), we obtain

$$\begin{aligned}\Sigma_{ij}^{kl}(\mathbf{r}) = \frac{g^2}{3}\Big[&\varphi_1\delta_{ij}\delta_{kl} + \varphi_2\delta_{i(k}\delta_{l)j} + \varphi_3\sum_{n=1}^{3}\delta_{in}\delta_{jn}\delta_{kn}\delta_{ln} \\ &+ 2\varphi_4\delta_{[ij}n_kn_{l]} + \varphi_5 n_{(i}\delta_{j)(k}n_{l)} + 2\varphi_6\delta_{[ij}\sum_{n=1}^{3}\delta_{kn}\delta_{ln]}n_n^2 \\ &+ 2\varphi_7\sum_{n=1}^{3}\delta_{[in}\delta_{jn}n_{(k}\delta_{l)n]}n_n + \varphi_8\sum_{n=1}^{3}\delta_{n(k}\delta_{l)(i}\delta_{j)n}n_n^2 \\ &+ \varphi_9 n_in_jn_kn_l + 2\varphi_{10}\delta_{[ij}\sum_{n=1}^{3}n_{(k}\delta_{l)n]}n_n^3 + \varphi_{11}\sum_{n=1}^{3}\delta_{n(i}n_{j)}\,\delta_{n(k}n_{l)}\,n_n^2\Big]\end{aligned} \tag{3.5}$$

where the functions $\varphi_i(\rho)$ are determined by the equations

$$\varphi_1 \equiv vf_1 + \Big[7m^2 - 6m + 2 - (m-1)^2\sum_{n=1}^{3}n_n^4\Big]f_2 + (64m^2 - 58m + 8)\,f_3$$

$$\varphi_2 \equiv 2(7f_1 + f_2 + 8f_3), \quad -\varphi_3 \equiv 3f_1 + f_2 + 6f_3$$

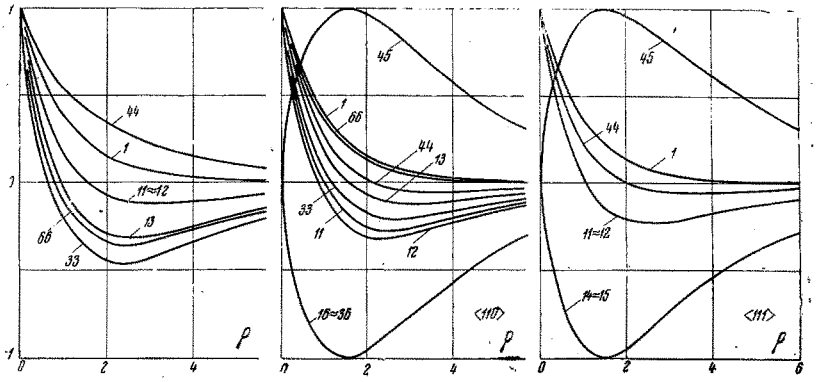

Fig. 4

$$\varphi_4 \equiv 2\,[(2m-1)f_2 + (11m-2)f_3], \quad -\varphi_5 \equiv 4\,(f_2+f_3)$$
$$\varphi_6 \equiv (m-1)\,(f_2+f_3) \qquad (3.6)$$
$$\varphi_7 \equiv 2\,(f_2+3f_3), \quad -\varphi_8 \equiv 4f_3, \quad \varphi_9 = 4\varphi_{10} = -\varphi_{11} \equiv 4f_2$$

It follows from expression (3.5) that for three characteristic directions [001], [110], and [111] the correlation matrix of the internal stresses is of the form shown in Fig. 1. Here the elements of the matrix which are different from zero are indicated by circles, those which are equal to zero are indicated by dots, and the elements of the same kind are connected by a line. In the first and third cases the correlation matrix Σ_{mn} is characterized by six different elements, and in the second, by nine.

As was to be expected, the symmetry of the matrix Σ_{mn} appears lower than the cubic symmetry, since together with the anisotropic distribution of the dislocation loops, in accordance with the cubic symmetry of the medium, the correlation matrix is characterized by the isolated direction in the expanse which connects two points between which there is a correlation connection.

Definite values of the components of the matrix Σ_{mn} can easily be found from the expression (3.5). For example, for the direction [001] we will have

$$\Sigma_{11}^{\circ} = \varphi_1 + \varphi_2 + \varphi_3, \quad \Sigma_{33}^{\circ} = \Sigma_{11}^{\circ} + \varphi_5 + \varphi_8 + 2\,(\varphi_4 + \varphi_6 + \varphi_7 + \varphi_{10})$$
$$2\Sigma_{66}^{\circ} = \varphi_2, \quad \Sigma_{12}^{\circ} = \varphi_1, \quad 4\Sigma_{44}^{\circ} = 2\varphi_2 + \varphi_5 + \varphi_8, \qquad (3.7)$$
$$\Sigma_{13}^{\circ} = \varphi_1 + \varphi_4 + \varphi_6 + \varphi_{10}, \quad \Sigma_{mn}^{\circ} \equiv \frac{3}{g^2}\Sigma_{mn}$$

As can be seen from expressions (3.5) and (3.7), the coordinate dependence of the component of the tensor Σ_{mn} is determined by the functions φ_i. Three of these, φ_1, φ_4, and φ_6, depend on the elastic moduli of the crystal, besides which the behavior of the function φ_1 depends on the direction along which the correlation connections are examined. Graphs of these functions are shown in Fig. 2. Curves 1, 2, and 3 are the relationships $\varphi_1(\rho)$ for aluminum, copper, and germanium (silicon) respectively. The functions $\varphi_i(\rho)$ are invariant relative to the axial properties and directions given in Fig. 3. Here the numbers show the index of the function φ_i.

As seen from the figures, all the functions φ_i have an extremum where $1.5 \leq \rho \leq 3$, after which they decrease with increase in the distance. The angular dependence of the function φ_1 is found to be weak. Hence, at distances of the order of the scale of the correlation, the difference in the functions φ_1 calculated for the directions [001], [110], and [111] is 1%. The dependence between the elastic moduli of the function φ_1 appears considerable at small ρ and practically disappears at distances somewhat exceeding the correlation scales, which is illustrated by curves 1-3 in Fig. 2. For functions $\varphi_4(\rho)$ and $\varphi_6(\rho)$ (curves 4 and 5) this dependence is negligibly small.

The coordinate dependences of the components of the correlation matrix of internal stresses, corresponding to the orientations [001], [110], and [111] of the section which connects the points between which the correlation connection is established, are given in Fig. 4. The numbers on the curves indicate the indices of the dimensionless correlation functions S_{mn}, obtained by means of division of the magnitude $\Sigma_{mn}(\mathbf{r})$ by $\Sigma_{mn}(0)$ in the case of $\Sigma_{mn}(0) \neq 0$ and by the absolute magnitude of the maximum (minimum) $\Sigma_{mn}(\mathbf{r})$ if

$\Sigma_{mn}(0) = 0$. For comparison, an exponent (curve 1) which describes the coordinate dependence of the binary correlation function of dislocation polarization is given on the graphs.

If we compare the curves for the [001] direction, we note that the monotonous decrease takes place only for the component S_{44}. In all the remaining cases the curves pass through a minimum which takes place at distances from $\rho = 2$ to $\rho = 3$.

As is seen from the structure of the matrix given in Fig. 1, for the direction [001] the changeover to cubic symmetry imposes additional conditions $S_{33} = S_{11}$, $S_{66} = S_{44}$, and $S_{13} = S_{12}$, whereby none of the components of the matrix S_{mn} is here converted to zero. On the other hand, for the direction [111], which is also characterized by six different components of the correlation matrix of internal stresses, the changeover to cubic structure imposes additional conditions $S_{14} = S_{15} = S_{45} = 0$. Corresponding to this, three of the six different components of the matrix S_{mn}, where $r \to 0$, return to zero. The [111] direction is also characterized by the fact that none of the components S_{mn} is described by a monotonous curve. We also point out that the components S_{14} and S_{15} are found to be negative, which indicates a noncorrespondence of the signs of random components of the longitudinal and shear components of the elastic fields. At the same time the correlations of the different shear components of the stresses, described by the component S_{45}, will be positive.

For the [110] direction the changeover to $r \to 0$ leads to conversion of the components $S_{16} = S_{26}$, S_{36}, and S_{45} to zero. Hence the function S_{45} in the whole region is positive, while the components $S_{16} \approx S_{36}$ are negative. The monotonous drop takes place for the component S_{66}, while the functions S_{11}, S_{12}, S_{33}, S_{13}, and S_{44} pass into the region of negative values at distances of the order of one or two scales of correlations and have a minimum at distances $2 \leq \rho \leq 3$, after which their asymptotic approach to the abscissa is observed.

For comparison of absolute values of correlation tensors of internal stresses we will introduce the parameter z_{mn}, which equals the ratio of $\Sigma_{mn}(0)$ to the value $\Sigma_{11}(0)$. Then, selecting aluminum as the definite case, we will find for all directions $z_{12} = 0.92$ and $z_{44} = 0.047$, that is, a correlation between the shear components of the field is considerably weaker than between the longitudinal components.

In order to compare the components Σ_{mn} which are converted to zero when $r \to 0$, we will introduce the coefficients ζ^{pq}_{mn}, having determined them as a ratio of the extreme value Σ^*_{mn} to the greatest of the magnitudes Σ_{pq}^* at a given ρ^*. All the functions S_{mn} have a maximum (minimum) at $\rho = 1.5$. The component Σ_{14} has the greatest of the maxima for the [111] direction, and the component Σ_{36} has the greatest of the maxima for the [110] direction. Accordingly, in the first case we find $\zeta^{14}_{15} = 0.76$, $\zeta^{14}_{45} = 0.045$, and in the second case $\zeta^{37}_{16} = 0.95$, $\zeta^{36}_{45} = 0.18$. Comparison of the values Σ_{14} for the [111] direction and Σ_{36} for [110] at the points of the minimum ($\rho = 1.5$) with corresponding magnitudes $\Sigma_{11}(\rho)$ shows that they are of one order: $\Sigma_{11}(1.5) \approx 2\Sigma_{44}(1.5)$ for the direction [111], and $\Sigma_{11}(1.5) \approx 2\Sigma_{36}(1.5)$ for the direction [110].

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